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Marcinkiewicz–Zygmund stability of quadrature-based Galerkin schemes

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ABSTRACT

We show that the Marcinkiewicz–Zygmund (MZ) condition provides a direct stability criterion for quadrature-based Galerkin discretizations, without requiring quadrature exactness. Under this condition, the discrete inner product induces a Riesz map that is spectrally equivalent to the continuous one, leading to well-posedness and stability of the resulting schemes, with error controlled by the MZ defect. The result is illustrated numerically on the sphere.

1. Introduction

Let $\Omega \subset \mathbb{R}^d$ be either the closure of a bounded connected open set, or a smooth compact manifold embedded in $\mathbb{R}^d$. Let $\mathbb{P}_n$ be a finite-dimensional function space on Ω, equipped with the L^2 inner product

$$\langle f, g\rangle = \int_\Omega f(x)g(x)\,dx,$$

and the associated norm $\|f\|_{L^2}^2 = \langle f, f\rangle$. For concreteness, one may think of $\mathbb{P}_n$ as the space of polynomials of degree at most n on Ω, or their restrictions in the manifold case.

In this setting, inner products on $\mathbb{P}_n$ play a central role in projection and variational formulations. Numerical discretizations based on polynomial approximation arise in two closely related settings: projection-based methods, such as Galerkin methods, and sampling-based approaches, such as hyperinterpolation [1,2]. In both cases, the evaluation of continuous inner products is required, and in practice these are approximated by numerical quadrature in the sense of

$$\langle p, q\rangle_m := \sum_{j=1}^m w_j p(x_j)q(x_j), \tag{1.1}$$

where $x_j \in \Omega$ are quadrature points and $w_j > 0$ are quadrature weights.

In classical numerical analysis (see, e.g., [3,4]), the quadrature rule is typically assumed to be exact for polynomials of sufficiently high degree, for instance up to degree $2n$ when $p, q \in \mathbb{P}_n$, since the integrand pq then belongs to $\mathbb{P}_{2n}$. In the univariate setting, for example, this requirement is naturally satisfied by Gauss quadrature rules. Under this assumption,

$$\langle p, q\rangle_m = \langle p, q\rangle, \qquad \forall p, q \in \mathbb{P}_n, \tag{1.2}$$

so that the discrete and continuous inner products coincide on the trial space, greatly simplifying both stability and error analysis. This coincidence is particularly useful in variational discretizations, where inner products enter both the bilinear form and the

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right-hand side. A prototypical example is the Galerkin approximation of a variational problem: find $u_n \in \mathbb{P}_n$ such that

$$a(u_n, v) = \langle f, v\rangle, \qquad \forall v \in \mathbb{P}_n,$$

where the bilinear form $a(\cdot,\cdot)$ is defined through the L^2 inner product, for instance $a(u, v) = \langle \mathcal{A}u, v\rangle$ for a given operator $\mathcal{A}$. In practice, one replaces $\langle\cdot,\cdot\rangle$ by its quadrature counterpart $\langle\cdot,\cdot\rangle_m$, leading to a fully discrete Galerkin scheme.

However, in many practical settings, such exactness is either unavailable or unnecessarily restrictive, especially when the sampling points are irregularly distributed or arise from data-driven processes. A more flexible alternative is provided by the Marcinkiewicz–Zygmund (MZ) condition (see, e.g., [5–7]), which ensures the norm equivalence

$$(1-\eta)\|p\|_{L^2}^2 \le \langle p, p\rangle_m \le (1+\eta)\|p\|_{L^2}^2, \qquad \forall p \in \mathbb{P}_n, \tag{1.3}$$

for some $\eta \in (0, 1)$ measuring the deviation between the discrete and continuous inner products.

Unlike exactness assumptions, the MZ condition depends only on the geometric distribution of the sampling points and is stable under perturbations, thereby naturally accommodating scattered and random data. Such inequalities have been established in a variety of settings, including scattered points on general manifolds [5] and random points on the sphere [8]. In particular, exact quadrature corresponds to the limiting case $\eta = 0$, so that the MZ framework recovers the classical theory as a special case. While the MZ condition has been extensively studied in approximation theory and sampling (see, e.g., [9–13]), and has also been incorporated into Nyström-type methods for integral equations in [14], its role in the numerical analysis of Galerkin discretizations remains largely implicit. The point of view adopted here is that stability is governed not by algebraic exactness of the quadrature rule, but by the geometry of the sampling set through the MZ condition. In this sense, the MZ condition provides a sampling-based stability criterion for Galerkin discretizations, allowing one to treat irregular or data-driven quadrature points within the same framework. This viewpoint complements the classical Galerkin analysis based on exact quadrature and can also be interpreted as a perturbation argument for the underlying bilinear form.

The contribution of the present note is to make this perturbation mechanism explicit in terms of the MZ defect and the associated discrete Riesz map, thereby linking the usual stability constants directly to sampling geometry. The main result below makes this observation precise. We prove that, under the MZ condition, the discrete Galerkin solution is well posed and approximates the continuous Galerkin solution with an error controlled by the MZ defect and the perturbation of the right-hand side.

Throughout the paper, C denotes a positive constant that may change from line to line, but is independent of the discretization parameters, the particular quadrature rule, and the functions being estimated, unless otherwise stated. The notation $A \lesssim B$ means that $A \le CB$ with such a constant C.

The paper is organized as follows. In Section 2, we discuss the MZ condition and establish the discrete Riesz map interpretation. Section 3 applies the result to Galerkin discretizations, and Section 4 presents a numerical experiment on the sphere.

2. MZ condition and discrete Riesz maps

Let $\{x_j\}_{j=1}^m$ be quadrature points with positive weights $\{w_j\}_{j=1}^m$. To analyze the effect of replacing the continuous inner product by its discrete counterpart, we introduce the associated discrete Riesz map.

Definition 2.1 (*MZ Condition*)**.** We say that the quadrature rule satisfies the *Marcinkiewicz–Zygmund* (MZ) *condition* on $\mathbb{P}_n$ if there exists $\eta \in (0, 1)$ such that (1.3) holds.

In contrast to exactness in the sense of (1.2), the MZ condition depends only on the geometric distribution of the sampling points and is stable under perturbations. In particular, it holds for a wide class of point sets, including structured designs as well as scattered or random sampling under suitable conditions.

Remark 2.1 (*Controlling the MZ Constant on the Sphere*)**.** On the sphere $\mathbb{S}^2$, the MZ constant η can be controlled in terms of the sampling density. Let

$$\delta_{\mathcal{X}_m} := \max_{x\in\mathbb{S}^2} \min_{x_j\in\mathcal{X}_m} \arccos\left(x \cdot x_j\right)$$

denote the mesh norm of the point set $\mathcal{X}_m = \{x_j\}_{j=1}^m$. It is known in [5,7] that the MZ condition holds provided the polynomial degree n satisfies $n \lesssim \eta/\delta_{\mathcal{X}_m}$. Since the mesh norm satisfies $\delta_{\mathcal{X}_m} \gtrsim m^{-1/2}$, this implies that the MZ condition holds when $m \gtrsim (n/\eta)^2$. Moreover, for equal-weight random sampling on $\mathbb{S}^2$, the MZ condition holds with high probability when $m \gtrsim n^2 \log n/\eta^2$; see [8]. These results show that the MZ defect η can be reduced by increasing the number of sampling points, providing a direct link between sampling density and the stability of quadrature-based discretizations.

We then formulate the MZ equivalence between the continuous and discrete L^2 inner products on $\mathbb{P}_n$, which will serve as a convenient tool for the analysis of quadrature-based Galerkin discretizations.

Definition 2.2 (*Discrete Riesz Map*)**.** Let $\langle\cdot,\cdot\rangle_m$ be the discrete inner product on $\mathbb{P}_n$. The *discrete Riesz map* $\mathcal{R}_n^{(m)} : \mathbb{P}_n \to \mathbb{P}_n$ is defined by

$$\langle \mathcal{R}_n^{(m)} p, q\rangle = \langle p, q\rangle_m, \qquad \forall p, q \in \mathbb{P}_n. \tag{2.1}$$

Throughout this section, operator inequalities and operator norms on $\mathbb{P}_n$ are understood with respect to the L^2-inner product $\langle\cdot,\cdot\rangle$.

Proposition 2.1. *Assume that the MZ condition holds on $\mathbb{P}_n$. Then the operator $\mathcal{R}_n^{(m)} : \mathbb{P}_n \to \mathbb{P}_n$ is symmetric positive definite and satisfies*

$$(1-\eta)I \le \mathcal{R}_n^{(m)} \le (1+\eta)I \tag{2.2}$$

in the L^2 inner product on $\mathbb{P}_n$, where I denotes the identity operator on $\mathbb{P}_n$. In particular, its condition number satisfies

$$\kappa(\mathcal{R}_n^{(m)}) \le \frac{1+\eta}{1-\eta}, \tag{2.3}$$

where $\kappa(\mathcal{R}_n^{(m)})$ is the condition number with respect to the L^2 inner product.

Proof. For any $p \in \mathbb{P}_n$, we have $\langle \mathcal{R}_n^{(m)} p, p\rangle = \langle p, p\rangle_m$. The MZ condition then yields $(1-\eta)\|p\|_{L^2}^2 \le \langle \mathcal{R}_n^{(m)} p, p\rangle \le (1+\eta)\|p\|_{L^2}^2$, which implies $(1-\eta)I \le \mathcal{R}_n^{(m)} \le (1+\eta)I$. □

Remark 2.2. The condition number $\kappa(\mathcal{R}_n^{(m)})$ quantifies the distortion between the discrete and continuous L^2 inner products on $\mathbb{P}_n$. In particular, the bound (2.3) shows that the MZ condition guarantees a uniformly well-conditioned identification between the two structures. As $\eta \to 0$, the discrete inner product approaches an isometry.

Lemma 2.1. *Assume that the MZ condition holds on $\mathbb{P}_n$. Then the L^2-induced operator norm of $\mathcal{R}_n^{(m)} - I$ from $\mathbb{P}_n$ to $\mathbb{P}_n$ satisfies*

$$\|\mathcal{R}_n^{(m)} - I\|_{\mathcal{L}(\mathbb{P}_n)} \le \eta, \tag{2.4}$$

where I denotes the identity operator on $\mathbb{P}_n$. Consequently,

$$|\langle p, q\rangle_m - \langle p, q\rangle| \le \eta\|p\|_{L^2}\|q\|_{L^2}, \qquad \forall p, q \in \mathbb{P}_n. \tag{2.5}$$

Proof. By Proposition 2.1, the operator $\mathcal{R}_n^{(m)}$ is self-adjoint on $\mathbb{P}_n$ with respect to the L^2 inner product and satisfies (2.2). Subtracting I gives $-\eta I \le \mathcal{R}_n^{(m)} - I \le \eta I$. Since $\mathcal{R}_n^{(m)} - I$ is self-adjoint, this implies (2.4). For any $p, q \in \mathbb{P}_n$, we have

$$\langle p, q\rangle_m - \langle p, q\rangle = \langle \mathcal{R}_n^{(m)} p, q\rangle - \langle p, q\rangle = \langle (\mathcal{R}_n^{(m)} - I)p, q\rangle.$$

Together with $|\langle p, q\rangle_m - \langle p, q\rangle| \le \|\mathcal{R}_n^{(m)} - I\|_{\mathcal{L}(\mathbb{P}_n)} \|p\|_{L^2}\|q\|_{L^2}$ and (2.4), we have the estimate (2.5). □

3. Stability of quadrature-based Galerkin discretizations

We now show that the MZ condition yields a direct stability estimate for quadrature-based Galerkin discretizations. Let $a(\cdot,\cdot)$ be a bilinear form on $\mathbb{P}_n$ satisfying

$$|a(u, v)| \le \beta\|u\|_{L^2}\|v\|_{L^2}, \qquad a(v, v) \ge \alpha\|v\|_{L^2}^2, \qquad \forall u, v \in \mathbb{P}_n, \tag{3.1}$$

for some $\alpha, \beta > 0$. We further assume that $a(\cdot,\cdot)$ admits the representation

$$a(u, v) = \langle \mathcal{A}u, v\rangle, \qquad \text{with } \mathcal{A}(\mathbb{P}_n) \subset \mathbb{P}_n.$$

Let $F \in \mathbb{P}_n^*$ and $F_m \in \mathbb{P}_n^*$ be bounded linear functionals. We consider the continuous variational problem: find $u_n \in \mathbb{P}_n$ such that

$$a(u_n, v) = F(v), \qquad \forall v \in \mathbb{P}_n, \tag{3.2}$$

and the quadrature-based discrete problem: find $u_n^{(m)} \in \mathbb{P}_n$ such that

$$a_m(u_n^{(m)}, v) = F_m(v), \qquad \forall v \in \mathbb{P}_n, \tag{3.3}$$

where $a_m(u, v) := \langle \mathcal{A}u, v\rangle_m$.

Theorem 3.1. *Assume that the MZ condition holds on $\mathbb{P}_n$. Then, for $\eta < \alpha/\beta$, the discrete problem (3.3) is well posed, and*

$$\|u_n^{(m)} - u_n\|_{L^2} \le \frac{1}{\alpha - \beta\eta}\Big(\|F_m - F\|_{\mathbb{P}_n^*} + \beta\eta\|u_n\|_{L^2}\Big). \tag{3.4}$$

Proof. Applying Lemma 2.1 to $p = \mathcal{A}u$ gives

$$|a_m(u, v) - a(u, v)| = |\langle \mathcal{A}u, v\rangle_m - \langle \mathcal{A}u, v\rangle| \le \eta\|\mathcal{A}u\|_{L^2}\|v\|_{L^2} \le \beta\eta\|u\|_{L^2}\|v\|_{L^2}.$$

Thus a_m is a bounded perturbation of a, and in particular $a_m(v, v) \ge (\alpha - \beta\eta)\|v\|_{L^2}^2$, so that a_m is coercive for $\eta < \alpha/\beta$. This implies that (3.3) is well posed. Let $e = u_n^{(m)} - u_n$. For any $v \in \mathbb{P}_n$, we have

$$a_m(e, v) = a_m(u_n^{(m)}, v) - a_m(u_n, v).$$

Using (3.2) and (3.3), this becomes

$$a_m(e,v) = F_m(v) - a_m(u_n, v) = (F_m - F)(v) + \big(a(u_n,v) - a_m(u_n,v)\big),$$

that is,

$$a_m(e,v) = (F_m - F)(v) + (a - a_m)(u_n, v), \qquad \forall v \in \mathbb{P}_n.$$

Taking $v = e$ and using the coercivity of a_m, we have

$$a_m(e,e) \geq (\alpha - \beta\eta)\|e\|_{L^2}^2.$$

On the other hand, $a_m(e,e) = (F_m - F)(e) + (a - a_m)(u_n, e)$. Therefore,

$$(\alpha - \beta\eta)\|e\|_{L^2}^2 \leq |(F_m - F)(e)| + |(a - a_m)(u_n, e)|.$$

The first term satisfies

$$|(F_m - F)(e)| \leq \|F_m - F\|_{\mathbb{P}_n^*}\|e\|_{L^2},$$

while the second term is bounded by

$$|(a - a_m)(u_n, e)| \leq \beta\eta\|u_n\|_{L^2}\|e\|_{L^2}.$$

Combining these estimates and dividing by $(\alpha - \beta\eta)\|e\|_{L^2}$ yields the estimate (3.4). □

Corollary 3.1 (*Continuous Right-Hand Side*). *Assume that*

$$F(v) = \langle f, v\rangle \quad \textit{and} \quad F_m(v) = \langle f, v\rangle_m$$

for some $f \in C(\Omega)$. Then under the assumptions of Theorem 3.1,

$$\|u_n^{(m)} - u_n\|_{L^2} \lesssim \frac{E_n(f)_\infty + \eta\|f\|_{L^2} + \beta\eta\|u_n\|_{L^2}}{\alpha - \beta\eta},$$

where $E_n(f)_\infty := \inf_{p\in\mathbb{P}_n}\|f - p\|_\infty$. Here the hidden constant is independent of f, u, and the quadrature rule.

Proof. For any $p_n \in \mathbb{P}_n$ and $v \in \mathbb{P}_n$, consider the following decomposition

$$\langle f, v\rangle_m - \langle f, v\rangle = \underbrace{\langle f - p_n, v\rangle_m}_{\text{discrete term}} + \underbrace{\big(\langle p_n, v\rangle_m - \langle p_n, v\rangle\big)}_{\text{MZ term}} + \underbrace{\langle p_n - f, v\rangle}_{\text{continuous term}}.$$

The MZ term is controlled by Lemma 2.1 in the sense of

$$|\langle p_n, v\rangle_m - \langle p_n, v\rangle| \leq \eta\|p_n\|_{L^2}\|v\|_{L^2}.$$

For the discrete term, using $f \in C(\Omega)$ and positive weights,

$$|\langle f - p_n, v\rangle_m| \leq \|f - p_n\|_{L^\infty}\sum_{j=1}^{m} w_j|v(x_j)| \leq C\|f - p_n\|_{L^\infty}\|v\|_{L^2},$$

where we used the Cauchy–Schwarz inequality and the MZ condition to bound the discrete norm of v. For the continuous term, the Cauchy–Schwarz inequality gives $|\langle p_n - f, v\rangle| \leq \|f - p_n\|_{L^2}\|v\|_{L^2}$. Combining these estimates yields

$$\|F_m - F\|_{\mathbb{P}_n^*} \leq C\Big(\|f - p_n\|_{L^\infty} + \|f - p_n\|_{L^2} + \eta\|p_n\|_{L^2}\Big) \leq C\Big(\|f - p_n\|_{L^\infty} + \eta\|p_n\|_{L^2}\Big).$$

Since $\|p_n\|_{L^2} \leq \|f\|_{L^2} + \|f - p_n\|_{L^2} \leq \|f\|_{L^2} + C\|f - p_n\|_{L^\infty}$, we obtain

$$\|F_m - F\|_{\mathbb{P}_n^*} \leq C\big(\|f - p_n\|_{L^\infty} + \eta\|f\|_{L^2}\big).$$

Taking the infimum over $p_n \in \mathbb{P}_n$ yields

$$\|F_m - F\|_{\mathbb{P}_n^*} \leq C\big(E_n(f)_\infty + \eta\|f\|_{L^2}\big).$$

Substituting this bound into Theorem 3.1 yields the desired estimate. □

4. Numerical experiment

We consider a simple model problem on the sphere to illustrate the theory, with the bilinear form

$$a(u,v) = \langle (I - \gamma\Delta_{\mathbb{S}^2})u, v\rangle, \qquad \gamma = 0.3,$$

restricted to the spherical polynomial space $\mathbb{P}_n$ of degree $n = 8$, with $\dim\mathbb{P}_n = (n+1)^2$. The exact solution $u_n \in \mathbb{P}_n$ is prescribed in the orthonormal real spherical harmonic basis, with randomly generated coefficients supported on the first few modes and normalized

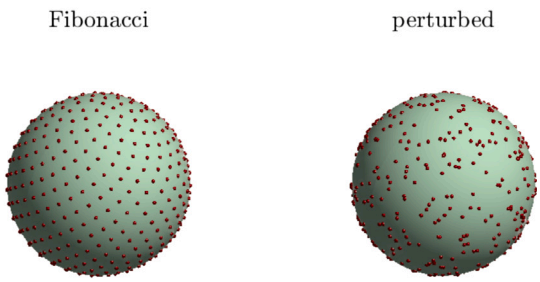

Fig. 1. Illustration of Fibonacci points and perturbed points with $m = 500$.

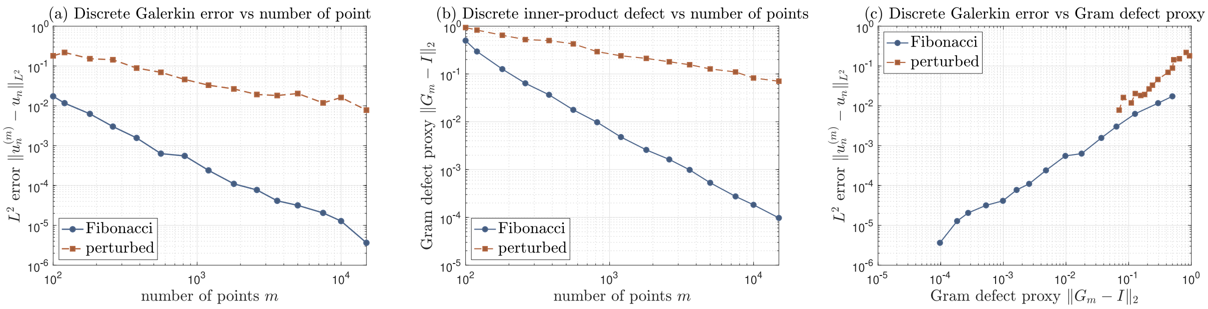

Fig. 2. Discrete Galerkin approximation on $\mathbb{S}^2$ with $n = 8$.

in L^2. The right-hand side is defined by $f = (I - \gamma \Delta_{\mathbb{S}^2})u_n$, so that u_n is the exact solution of the continuous problem. The discrete inner product is approximated by equal-weight quadrature,

$$\langle p, q \rangle_m = \sum_{j=1}^{m} w_j p(x_j) q(x_j), \qquad w_j = \frac{4\pi}{m},$$

using two families of sampling points, illustrated in Fig. 1: (i) spherical Fibonacci points, and (ii) perturbed Fibonacci points obtained by random perturbations followed by projection onto $\mathbb{S}^2$. For each

$$m \in \{100, 120, 180, 260, 380, 560, 820, 1200, 1800, 2600, 3600, 5000, 7500, 10000, 15000\},$$

we assemble the discrete Gram matrix

$$G_m = \left(\langle \phi_i, \phi_j \rangle_m\right)_{i,j=1}^{N},$$

and use $\|G_m - I\|_2$ as a computable proxy for the MZ defect. Since the spherical harmonics used here form an orthonormal basis of $\mathbb{P}_n$, the matrix G_m is precisely the matrix representation of the discrete Riesz map $\mathcal{R}_n^{(m)}$ in this basis. Hence $\|G_m - I\|_2$ coincides with the L^2-induced operator norm of $\mathcal{R}_n^{(m)} - I$, and therefore provides a computable proxy for the MZ defect. The discrete Galerkin solution $u_n^{(m)}$ is computed accordingly, and the error is measured in coefficient space, which coincides with the L^2 norm.

Fig. 2 reports the following: (a) the error $\|u_n^{(m)} - u_n\|_{L^2}$ versus m; (b) the Gram defect proxy $\|G_m - I\|_2$ versus m; and (c) the error versus $\|G_m - I\|_2$. The results are fully consistent with the theoretical prediction. As the number of points m increases, both sampling families lead to a decay of the Gram defect $\|G_m - I\|_2$, indicating that the discrete inner product converges to the continuous one. For the structured Fibonacci points, this decay is faster, reflecting their more uniform distribution. For the perturbed points, the decay is slower but remains clearly observable, showing that the MZ condition continues to improve with increasing sampling density even under irregular perturbations. Most importantly, panel (c) reveals an approximately linear relationship on the log–log scale between

$$\|u_n^{(m)} - u_n\|_{L^2} \quad \text{and} \quad \|G_m - I\|_2,$$

for both sampling families. This provides direct numerical evidence that the MZ defect serves as an effective predictor of the discretization error, in agreement with the stability estimate in Theorem 3.1.

These results demonstrate that the accuracy of the quadrature-based Galerkin solution is governed primarily by the distortion of the discrete inner product, and that the proposed framework remains effective even for non-ideal, perturbed sampling configurations.

5. Concluding remarks

We have shown that the MZ condition provides a direct criterion for the stability of quadrature-based Galerkin discretizations, without requiring quadrature exactness. The analysis reveals that the discrete inner product induces a uniformly bounded perturbation of the continuous L^2 structure, leading to well-posedness and stability with constants explicitly controlled by the MZ defect. This behavior is confirmed by the numerical experiments on the sphere. The framework suggests possible extensions to more general operators and a promising direction for the analysis of sampling-based discretizations under non-structured data.

Data availability

Data will be made available on request.

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